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CAS in Australia: A Brief Perspective

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In recent years in Australia, CAS has been incorporated successfully into the curriculum and pedagogy of a steadily increasing but still relatively small number of mathematics classrooms. This paper provides a brief overview of how CAS has influenced the examination assessment of mathematical knowledge. It also describes changes to teacher practice and how students use CAS as seen by one teacher's experiences during the implementation of a CAS-active mathematics subject in Victoria, Australia. More generally, teachers in Victorian CAS-active classrooms are challenged as to how to incorporate the symbolic capabilities of CAS in teaching and learning and by what role by-hand skills play in CAS-active learning environments. These challenges that confront teachers should help frame future CAS research and its effects on curriculum, instruction and assessment.

Introduction

Geographically, Australia consists of six states and two territories. Each Australian state is responsible for developing and implementing its own mathematics curriculum and assessment of student mathematical knowledge. Policy regarding what technology is recommended for use in the classroom, what technology is permitted in assessment and how pre-university entrance examination assessment is conducted varies considerably amongst the six Australian states. Over the last three decades, Victoria, one of the six Australian states, has made a successful transition from allowing its secondary school mathematics students to use scientific calculators to encouraging and supporting secondary school mathematics teachers and students to use graphics calculators in the mathematics classroom. One major educational benefit of allowing mathematical tools such as graphics calculators in mathematics education has been that problems/demonstrations that could only previously be solved/performed *in principle* before the advent of graphics calculators can now be solved/performed *in practice* with a graphics calculator. This has assisted in widening the variety of mathematical experiences shared between secondary school teachers and students and offered our students greater opportunities to solve more realistic mathematical problems, undertake more mathematical modelling and help to understand the role that parameters have in changing the behaviour of mathematical functions.

After several years of graphics calculators being successfully incorporated in classrooms and in assessment, Victorian mathematics education is now moving towards a full implementation of CAS-active mathematics subjects in upper secondary school (scheduled for commencement in 2010). The first step taken to achieve this goal was to develop a CAS-active pre-university entrance mathematics subject. In Victoria, The University of Melbourne in conjunction with the Victorian Curriculum and Assessment Authority (VCAA) and industry partners, conducted a three-year research project (2000-2002) entitled 'Computer Algebra Systems in Schools-Curriculum, Assessment and Teaching (CAS-CAT Project)'. The CAS-CAT Project investigated changes that regular access to CAS might have on senior secondary mathematics teaching and learning,

including formal assessment (HREF1). The project explored the feasibility of offering new mathematics subjects that incorporate CAS in the classroom and in timed examination assessment. Integral to the CAS-CAT Project, students from three pilot schools studied an accredited pilot CAS-permitted subject, Mathematical Methods (CAS).

Mathematical Methods (CAS) involves the study of functions, calculus, algebra and probability distributions (discrete and continuous) and is conducted in parallel to an existing graphics calculator-permitted subject, Mathematical Methods. Both subjects are very similar in mathematical content with Mathematical Methods (CAS) offering increased opportunities for general analysis of a wider range of mathematical functions and applications. Each school in the pilot program was allocated a different brand of CAS (TI-89, CASIO FX 2.0 and HP 40G) for their students to use. At the completion of their two year study, 78 students sat for two CAS-permitted examinations as part of their final school assessment (required for university entrance).

Following the initial pilot phase, an increasing number of schools (with a wider range of permitted CAS calculators and computer-based software) have offered Mathematical Methods (CAS). See Table 1 below for student enrolment trends for the period 2002-2006. From 2006, the structure of the two written examinations has changed to a technology-free examination paper and a technology active examination paper. Further content, administrative details, examination papers and assessment reports can be accessed from the Victorian Curriculum and Assessment Authority (VCAA) website (HREF2 and HREF3).

Year	2002	2003	2004	2005	2006
Students	78	269	389	333	539

Table 1: Mathematical Methods (CAS) Student Enrolments (2002-2006).

Victoria is the only Australian state that permits the use of handheld CAS calculators and computer-based CAS software in the teaching, learning and external examination assessment of pre-university entrance mathematics.

One Examination System's CAS Response: A Snapshot

Policy adopted by some examination systems to allow CAS use in their examinations has provoked much debate about what mathematical knowledge can be assessed. While different examination systems have adopted a variety of technology policies, at the core of the debate is the suitability of traditional examination questions that have aimed to test knowledge of symbolic manipulative skills and procedures. The symbolic capabilities of CAS can affect these traditional examination questions by 'gobbling' all or a substantial number of intermediate steps (Flynn & Asp, 2002), creating shifts in the mathematical knowledge assessed (Kokol-Voljc, 2000) or creating difficulties in setting fair questions that test symbolic reasoning and connections (Flynn & Asp, 2002). Such technical and epistemological difficulties were foreshadowed by Monaghan (2000) who concluded that it is generally harder to set examination questions that encourage CAS use compared to setting examination questions that bypass or exclude CAS use. Cognisant that changes to examination assessment must occur, Stacey (2002) argues that assessing mathematical knowledge primarily through calculation with CAS is ultimately untenable. Permitting

CAS use in examinations calls for a shift away from testing routine procedures and a greater emphasis placed on assessing the formulation and interpretation of mathematical models. However a reduction in testing routine procedures could increase the level of difficulty because fewer ‘easy’ marks would be available to candidates across a wide range of aptitudes (Berry, Johnson, Maull & Monaghan, 1999). This brief discussion highlights some of the challenges faced by examination panels to construct fair examination questions that assess valued mathematical knowledge with CAS and enhance learning back in classrooms.

Mathematical Methods (CAS) examination papers have now been set in Victoria since the first CAS-permitted examinations were conducted in 2002. This five year period of CAS-permitted examination assessment, where a widening range of approved CAS devices can be used, has provided an opportunity to detect trends in the types of examination questions set. VCAA Assessment Reports detailing student performances on these examinations have reported consistently that the symbolic facility of CAS was used quite well and that no discernable advantage was seen by the assessors of one CAS over another, although in some cases techniques for dealing with the mathematics involved varied according to the CAS. This illustrates that the need to make an equitable examination exerts a significant effect on examination question design. Inequities between different CAS can be offset largely by making a wide variety of solution methods available. A clear understanding of the range of effects CAS may have on question design is required so examination panels can strategise accordingly to develop sound assessment that supports effective mathematics education.

In the period 2002-2006, Mathematical Methods (CAS) examination papers have responded to the influence of CAS by setting examination questions that:

1) are within normal by-hand capabilities;
2) require numerical/graphical techniques for their solution;
3) could involve problematic CAS use and are best attempted without CAS;
4) contain parameters and require algebraic reasoning for their solution.

The various responses to allowing CAS in the examinations have an influence on the doing and learning of mathematics back in the classroom. For example, Flynn (2003) reports that an increased access to a wider range of solution methods has implications for what solution methods a teacher will privilege in the classroom.

(1) Examination questions set within normal by-hand capabilities

Some examination questions are deliberately set well within student mental strategies or by-hand algebraic skills and could be answered most efficiently without CAS. The setting of these examination questions indicates that the testing of rehearsed procedures is still considered important by this education system. With CAS, such examination questions generally test recognition of the required CAS capability, correct entry of expression and correct use of syntax (see Figure 1). This illustrates how CAS can shift the nature of what an examination question is testing (Kokol-Voljc, 2000).

While CAS can significantly improve accuracy and reliability of some symbolic computations (routine examination questions), examination questions deemed to be 'trivialised' by CAS may not be so. For example, *only* 70% of the CAS cohort correctly identified the linear factors of $x^4 + x^3 - 3x^2 - 3x$ compared to 55% of students using a graphics calculator (see Figures 1 and 2). A common error made by the CAS cohort

(either with or without CAS) was to factorise over the rational field (17% of CAS students chose option A) rather than over the real field.

The **linear** factors of $x^4 + x^3 - 3x^2 - 3x$ over R are

A. $x, x + 1, x^2 - 3$

B. $x, x + 1, x + \sqrt{3}, x - \sqrt{3}$

C. $x, x + 1$

D. $x + 1, x + \sqrt{3}, x - \sqrt{3}$

E. $x + 1, x^3 - 3x$

Figure 1: VCAA 2002 Mathematical Methods (CAS) Examination 1, Part I, Question 9.

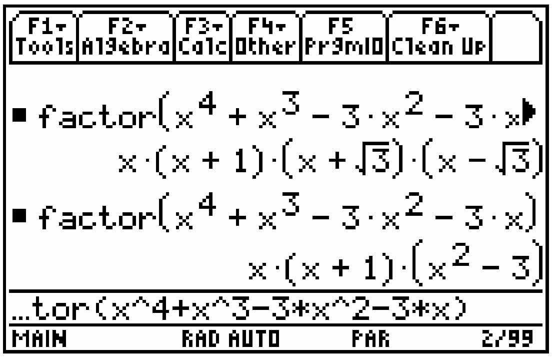


Figure 2: TI-89 screenshot showing a correct and incorrect response to question 9.

(2) **Numerical/graphical questions (CAS no advantage over graphics calculators)**

One noticeable effect CAS has on mathematical activity is its potential to widen the number of possible solution methods available. This 'explosion of methods' (Artigue, 2002) is a function of CAS's numerical, graphical and symbolic representations and the various capabilities that exist within these representations. Flynn & McCrae (2001) discussed the possibility of setting examination questions that could be answered by using numerical or graphical techniques only and therefore not advantage a CAS user over a graphics calculator user. Figures 3 and 4 illustrate an examination question part and its solution where a CAS user is deemed to have no advantage over a graphics calculator user i.e. students use CAS like a graphics calculator.

On an adventure park ride, riders are strapped into seats on a platform which starts 15 metres above the ground and goes up and down. During the first 60 seconds of the ride, the distance, y metres, of the platform above the ground, t seconds after the ride starts, can be modelled by the formula $y(t) = 15 + e^{0.04t}\sin\left(\frac{\pi t}{3}\right),\ 0 < t \le 60$.

According to this model, the platform is exactly 6 metres above the ground for the first time about 58 seconds into the ride.
Find this time correct to two decimal places of a second**.**

Figure 3: VCAA 2002 Mathematical Methods (CAS) Examination 2, Question 4(b).

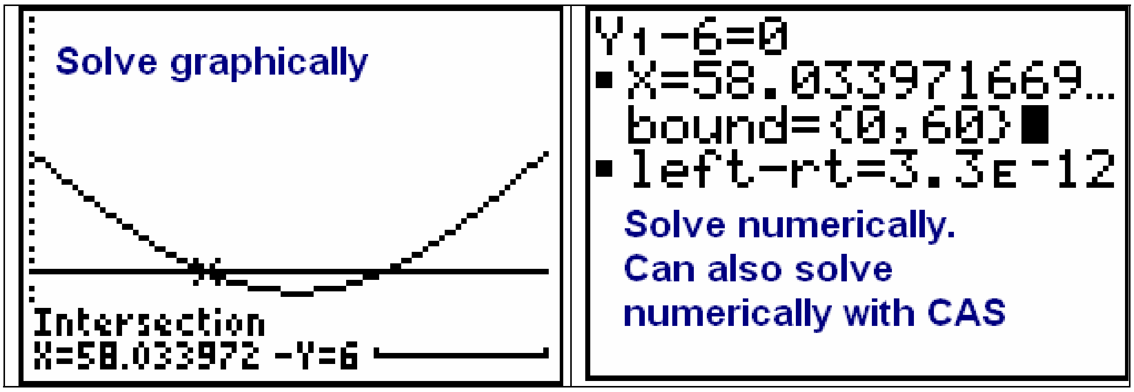


Figure 4: TI-84 solution screenshots to question 4(b).

These examination question types are best solved using numerical or graphical solution techniques and they have played a prominent role in testing student mathematical knowledge in this examination system. It is important to assess applications of algebra and calculus techniques in context and within multiple representations.

(3) **Problematic CAS use**

In this examination system, examination questions have been set where direct CAS use could be problematic and are probably best attempted without the use of CAS. Figures 5 and 6 illustrate such an examination question and a CAS screenshot showing a 'puzzling' output involving the signum function i.e. $\text{sign}(x)=\begin{cases}1 & \text{if } x>0\\ -1 & \text{if } x<0\end{cases}$. 53% of students chose option B while only 31% correctly selected option A. Students using CAS need to be able to reconcile the form of an output.

If $y=\left|\sin(x)\right|$, the rate of change of y with respect to x at $x=k$, $\pi<k<2\pi$, is

A. $-\cos(k)$ **31%**

B. $\cos(k)$ **53%**

C. $-\sin(k)$ **6%**

D. $\sin(k)$ **5%**

E. $k\cos(1)$ **5%**

Figure 5: VCAA 2004 Mathematical Methods (CAS) Examination 1, Part I, Question 19.

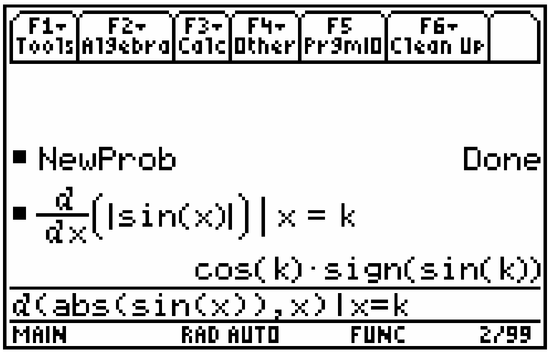


Figure 6: TI-89 CAS screenshot showing the signum function.

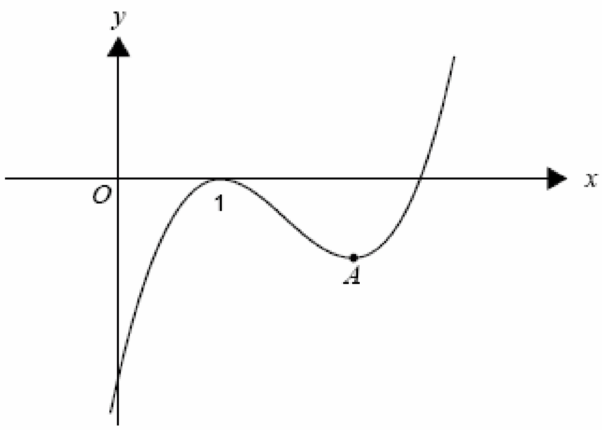
Such examination questions illustrate that in this examination system, an increased teaching and learning emphasis must be placed on:

- developing student algebraic insight (see Pierce and Stacey framework);
- providing opportunities for reflection of CAS outputs and identification of equivalent forms either mentally or with pen and paper approaches.

(4) Parameters

A prominent response made by this examination system has been to set examination questions and question parts involving functions expressed in terms of parameters (see Figures 7 and 8 below).

The graph of $y = (x - 1)^2(x - a)$ where $a > 1$ is shown below.



Find the exact value of a such that the local minimum at point A lies on the line with equation $y = -4x$.

Figure 7: VCAA 2004 Mathematical Methods (CAS) Examination 2, Question 1(f).

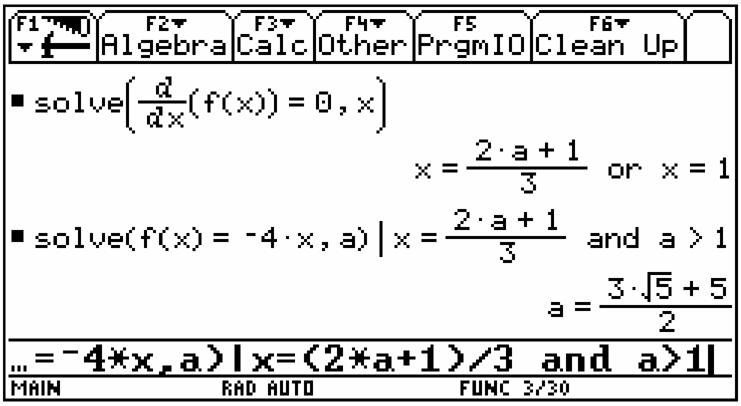


Figure 8: Voyage 200 CAS solution screenshot to question 1(f).

Such examination questions illustrate that in this education system, an increased teaching and learning emphasis must be placed on:

- parameters and how they affect the behaviour of a function;
- students understanding and carrying out solution plans with CAS;
- formulation, knowledge of algebraic structure, equivalent forms and symbolic reasoning involving parameters.

CAS and Teaching: One Australian Teacher's Experience

Much CAS research has been conducted by Australian researchers over the last decade (HREF1). The intention here is report briefly on what is happening in CAS-active classrooms rather than reporting upon the themes and findings of Australian CAS research. Currently, a very small number of Victorian teachers teaching in CAS-active classrooms have documented their experiences in state/national conference papers or mathematics education journal articles. As such, it has been decided to share the experiences of one experienced Victorian mathematics teacher, David Tynan, in his implementation of teaching with CAS.

Tynan (2003) reflected on his teaching experiences with CAS and categorised different student mathematical behaviours when they had unfettered access to CAS. These different behaviours influenced how effectively a student used CAS in his classroom and in internal and external assessment. Tynan did this by using caricatures (exaggerated representations of a person's prominent features) to list some of the common ways students responded to the introduction of CAS and describes the different needs of students in a CAS-active classroom. Importantly, Tynan notes that his representations of students exaggerate their most notable characteristics and that no one student would *always* behave solely in the manner described. These profiles are meant to depict ways in which students react to doing and learning mathematics with a new technology. The four student caricatures presented by Tynan are:

1) The sceptics
2) The technocrats
3) The dependents
4) The battlers

Each caricature is described briefly below.

(1) The Sceptics

The sceptics initially expressed concerns that using CAS would erode their proficiency for performing traditional algorithms accurately and efficiently. While they did not use CAS as frequently as their peers, their maintained scepticism and determination to master by-hand routines provided a strong basis for their eventual development as effective CAS users i.e. recognising when CAS use would be more efficient. The sceptics were generally quite proficient at interpreting CAS outputs correctly. These students needed proof that some procedures are completed more efficiently with CAS and that not all problems can be solved by analytic means.

(2) The Technocrats

The technocrats relished using CAS through the discovery of new solution approaches such as 'nesting' different CAS commands. They were also interested in the simplification conventions and syntax used by CAS. These students experienced some difficulties reconciling CAS outputs, recognising equivalent algebraic forms and documenting their solution approaches in written communication. Despite these difficulties, the technocrats regularly developed inventive ways to recognise equivalent algebraic expressions. The technocrat student needed encouragement to use by-hand approaches more frequently to solve problems. This was in order to be able to make greater sense of CAS outputs and not to become overly dependent on CAS.

(3) The Dependents

These students placed great faith in CAS to help solve problems they normally would experience algebraic difficulties with. CAS assisted these students to perform routine algebra and calculus-based techniques. However, they experienced difficulties solving more challenging problems requiring formulation, comprehending CAS outputs and recognising equivalent forms. Frequently, they used CAS syntax in solution documentation and were not discerning or efficient CAS users. Tynan notes that their insistence on completing all new algorithms with CAS was particularly challenging. The dependent student needed to spend more time reflecting on what answer they might expect from CAS use. By directing them to not CAS as often, he believes that these students developed better manipulative skills.

(4) The Battlers

The battlers typically exhibited a negative attitude towards CAS and generally managed to perform most basic routines with CAS. However, they experienced problems when the use of CAS required flexibility in mathematical thinking. These students required increased attention to help consolidate their basic algebraic skills.

In his CAS experience, Tynan notes that the sceptics and the technocrats were closest to his philosophical and pragmatic response to teaching with CAS. Over time however, he believes that his teaching approach mirrored the behaviour of the sceptics more closely. As a result, he concluded that many students in his class became more discerning and effective CAS users.

Importantly for beginning teachers, Tynan recorded possible difficulties that some students encountered while learning to use CAS effectively. These difficulties included:

- Working with variables and defined functions
- Order of operations
- Error messages
- Effective use of the symbolic manipulator
- Remembering syntax
- Equivalence of form
- Recording work

Tynan notes that through a combination of his subtle reminders and frequency of student use, eradicating these technical and mathematical difficulties was not insurmountable and occurred at different learning rates. The observations made by Tynan should prove invaluable for teachers beginning to teach mathematics with a CAS and also frame ideas for future research strands.

CAS and Textbooks

To coincide with the introduction of newly accredited mathematics subjects, most mathematics textbook publishers in Victoria produced a textbook that incorporated CAS in some way. Typically, CAS was incorporated through the inclusion of syntax/navigation tips for CAS use, screenshots of CAS devices complementing by-hand solutions to worked examples, questions and tasks that require CAS use for their solution. In cases where corresponding teacher editions were published, strategies and advice for implementing CAS in lesson situations were included.

Some excerpts illustrating how CAS has been incorporated into one such set of textbooks, *MathsWorld Mathematical Methods Units 1 & 2* and *MathsWorld Mathematical Methods Units 3 & 4* (Macmillan Education Australia, 2006), are shown in Figures 9, and 10 below.

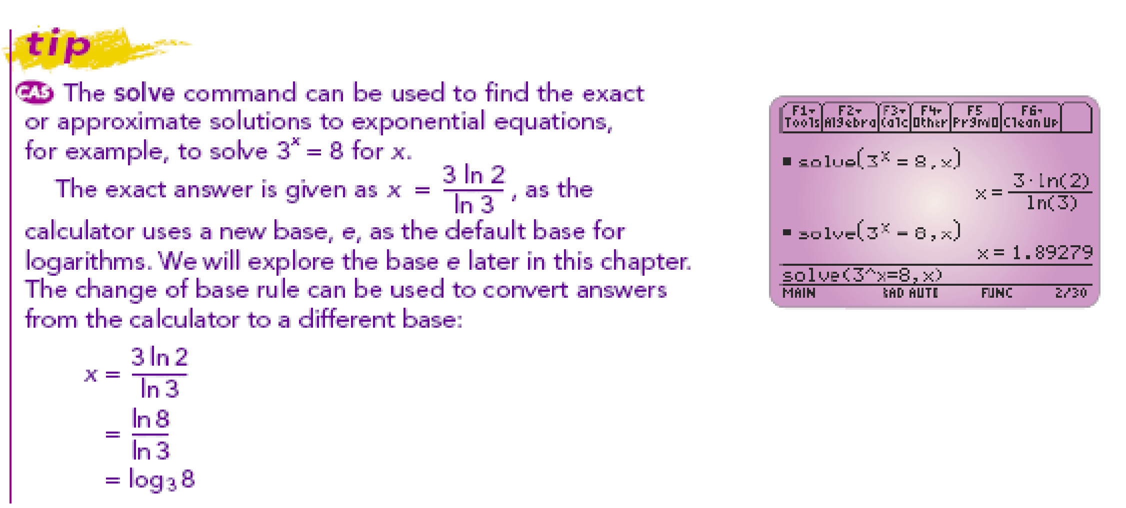


Figure 9: Textbook excerpt showing a TI-89 CAS solution tip.

Example 1

Solve the following simultaneous linear equations using technology.

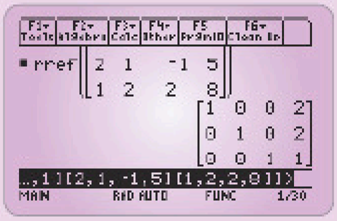
$$\begin{aligned} x - y + z &= 1 \\ 2x + y - z &= 5 \\ x + 2y + 2z &= 8 \end{aligned}$$

Solution

There are a number of ways of solving these equations.

Method 1

First create a matrix consisting of the numerical values in the equations on both sides of the equals sign:



$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 2 & 1 & -1 & 5 \\ 1 & 2 & 2 & 8 \end{bmatrix}$$

Then use the **rref** procedure introduced in section 3.4.

Read off the solution: $1x + 0y + 0z = 2$, that is, $x = 2$. Similarly $y = 2$ and $z = 1$.

As a check, substitute into the first equation: LHS = 2 – 2 + 1 = 1 = RHS.

Method 2

Use **solve** as shown in the tip after example 3, section 3.4.
The syntax is **solve(eqn1 and eqn2 and eqn3,{x,y,z})**.

Read off the solution: $x = 2$, $y = 2$, $z = 1$.

(The method readily extends to more than three variables.)

Figure 10: Textbook excerpt showing TI-89 CAS solution methods.

Challenges and Future Directions

In assessment, future work must focus on widening the bandwidth of mathematical knowledge assessed and improving the testing of symbolic reasoning. In the Victorian examination system, there has been a relatively strong emphasis placed on testing mathematical knowledge through numerical/graphical CAS use and rehearsed symbolic procedures. This has meant that prominent uses for CAS back in the classroom include students using numerical/graphical approaches to solving appropriate problems and using CAS to compensate for inadequate algebraic skills. However, we must ensure that we do not just use CAS to replicate by-hand problems but to also set new and interesting problems.

A big challenge facing Victorian teachers teaching with CAS is how to successfully integrate the symbolic capabilities of CAS to enhance the teaching and learning of algebra and calculus? A second related challenge is to determine what roles by-hand skills play in learning mathematics in a CAS-active environment? Tynan, for example, found that sound knowledge of by-hand algebra skills can play an important role in developing a good understanding of CAS outputs. These two challenges cause concern for many Victorian teachers and should frame future CAS research.

Disclaimer

The views and analysis expressed in the CAS and assessment section of the paper are not necessarily shared by the Victorian Curriculum and Assessment Authority, the statutory government body that oversees curriculum and assessment of Mathematical Methods (CAS) in Victoria, Australia.

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